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Philosophy of Math

May 10, 2023

Hume's Pirahã Problem

Neo-logicism is a modern approach to the philosophy of mathematics that traces its roots to 19th century logician Gottlob Frege. Neo-logicism has two (greatly appealing) distinctive philosophical commitments: [1] mathematical objects are actual objects that really exist and [2] all mathematical truths are built out of logical truths. The chosen method for many to achieve these results are abstraction principles. Abstraction principles take the form $§\alpha = §\beta \leftrightarrow \alpha \sim \beta$. They relate, on the left hand, 'thin' objects corresponding to α and β to, on the right hand, a designated unity relation '~' held by α and β. From this principle, the neo-logicist purports, by recognizing that designated relation, we inherently also recognize those thin objects.

Prime among these abstraction principles is "Hume's Principle" (HP). HP states that $\#\alpha = \#\beta \leftrightarrow \alpha \approx \beta$. In words, HP states that the number of αs is equal to the number of βs iff α and β are equinumerous, i.e. for every α there is a β. This principle is crucial to the neo-logicist because it satisfies commitment [1] by grounding ordinal numbers as thin objects and satisfies commitment [1] by what is known as Frege's Theorem. Frege's Theorem states that given HP and second-order logic, all of Dedekind-Peano Arithmetic can be derived. As a result, HP is widely regarded as the most foundational abstraction principle; in addition, it is one of the few well-behaved abstraction principles (for demonstration of this, see Cook Forthcoming). HP's status as a true principle is thus crucial to the neo-logicist project. If false, it would undermine a large portion of mathematics neo-logicism can thus far justify and it would cast doubt on the prospects of other less well-behaved abstraction principles.

In this paper, I will analyze the two prominent understandings of abstraction principles, determining on what grounds an abstraction principle may be false or fails to completely justify commitments [1] and [2]. Then, I will present the case of the Pirahã tribe, an indigenous tribe from the Amazon that current studies suggest lack quantificational and number words (even for the number one). The task will be to decide whether the Pirahã are a legitimate counterexample to HP based on the criteria I set out in §2 and §3. Finally, a few objections against the both studies surrounding the Pirahã language and the efficacy of the counterexample I provide are considered. In total, I outright reject the first view of abstraction principles and conclude that the remaining neo-logicist program requires a far more detailed epistemological story to be satisfactory.

1 | Thin Objects

However, before embarking on the roadmap I set out, it's first important to understand the nature of the mathematical objects both views of abstraction principles share, namely, that mathematical objects are "thin objects." Øystein Linnebo in his book *Thin Objects: An Abstractionist Account* frames thin objects in relation to "thick" objects such as tables, chairs, or other physically instantiated objects. As Linnebo describes, objects are thin inasmuch as they do "not make a substantial demand on the world," i.e. the conditions for a thin object's existence are very minimal. Specifically, Linnebo claims that that which is a "possible referent of a singular term" is an object. Notably, this is not a definition of what an object is (hence the clunky inverted phrasing); it's merely a sufficient condition for objecthood. This condition allows for the assignment of objecthood to a broad spectrum of things—including both traditional thick objects and thin objects—via a "criterion of identity," i.e. what identifies an object as this or that specific object.

Thus, a thin object is a possible referent of a singular term specified by a criterion of identity which does not make substantial demands for the object's existence. Particularly, specification for thin objects typically takes the form of abstraction principles.

2 | Symmetrical Abstraction Principles

There are two views on how abstraction principles actually specify and thus refer to the thin objects. The standard view of abstraction principles—the view outlined in the introduction—holds that the two sides of an abstraction principle "recarve" the same content. The favorite toy example of both Frege and Linnebo demonstrates this well: directions. Frege defines the abstraction principle $d(l_1) = d(l_2) \leftrightarrow l_1 \parallel l_2$: the direction of line 1 (l_1) is the same as the direction of line 2 (l_2) iff l_1 and l_2 are parallel. For Frege, to say that $l_1 \parallel l_2$ is the exact same as saying $d(l_1) = d(l_2)$. This view is symmetrical in that the two sides of the principle are taken to be "made true by precisely the same facts or states of the world" (*Thin Objects*). Meaning, there is nothing described in one of the sides that is not described in the other.

It's rather clear for most abstraction principles, and especially HP, that the biconditional is true. $\#\alpha = \#\beta$ is true only in case $\alpha \approx \beta$ is also true, and vice versa. But the sort of talk Frege and neo-logicists engage in implicates a secondary condition for abstraction principles, namely $\mathcal{K}(\#\alpha = \#\beta) \leftrightarrow \mathcal{K}(\alpha \approx \beta)$, where $\mathcal{K}$ is the predicate for knowledge. This is evident for a variety of reasons. First, abstraction principles are as much about developing an epistemological story about how we know about thin objects as they are about metaphysically grounding thin objects. In fact, Frege motivates abstraction principles by asking "How, then, are numbers given to us, if we cannot have an ideas or intuitions about them?" (73). Thus, if somehow this second biconditional fails, the neo-logicist loses their ability to know that mathematical objects really truly exist, violating commitment [1]. But this isn't the only evidence: Second, because the symmetric view claims that there is nothing over and above on either side of the abstraction principle, to know one side must equate to knowledge of the other side. This cannot be sidestepped. We cannot say it's possible to know one side and not know another because we lack the concepts to derive the second side; as stated above, abstraction principles are supposed to be the machinery by which we obtain the concepts of mathematical objects. To sidestep in this manner would beg the question.

The necessity of this second biconditional for the neo-logicist project places it under direct empirical scrutiny. If there exists a non-problematic case where someone knows that two collections of things are equinumerous, but does not know that the two collections have the same number, it is a legitimate counterexample. To prove this counterexample, it is sufficient to show that that person understands equinumerosity, but fails to understand numbers because equinumerosity is supposed to give conceptual access to numbers.

3 | Asymmetrical Abstraction Principles

In contrast to the symmetric view, the asymmetric view only supposes that $\alpha{\sim}\beta \Rightarrow \S\alpha = \S\beta$ and not the reverse. The use of the double struck arrow ⇒ is intended to signify a narrowly defined "sufficiency" operator. This definition comes from Linnebo's defense of the asymmetric view; he defines four "job descriptions" for the sufficiency operator. They will be described in depth as needed, but briefly, they are (A) the Ontological Expansiveness Constraint, (B) Face Value Constraint, (C) Epistemic Constraint, and (D) Explanatory Constraint. Note that though the sufficiency operator was introduced in support of the asymmetric view, it can be used for the symmetric view (which Linnebo does at times).

On this view, there is something more described on the $\S\alpha = \S\beta$ side than the $\alpha \sim \beta$ side, namely, the thin objects. That is to say, the asymmetric view denies the symmetric claim that abstraction principles simply "recarve" the same content; rather, $\alpha \sim \beta$ is said to "metaphysically grounds" or "metaphysically grounds" $\S\alpha = \S\beta$. The intuition is that only $\S\alpha = \S\beta$ describes thin objects and thus makes a more substantial demand than the other side. This behavior is contained under constraint (A), the Ontological Expansiveness Constraint. However, the specifics aren't relevant here.

The important piece is that Linnebo formally specifies the epistemic consequences of the asymmetric view. He makes two claims: Given two statements Φ and Ψ,

> "**Epistemic Constraint** [constraint C]
>
> "If Φ ⇒ Ψ, then it is possible to know Φ → Ψ; and if additionally Φ is known, then this possible knowledge is compatible with continued knowledge of Φ" (*Thin Objects).*

and

> "If Φ ⇒ Ψ and $\mathcal{K}(\Phi)$, then $\Diamond\mathcal{K}(\Psi)$" (◇ represents metaphysical possibility) ("Précis").

I'm primarily concerned with his second claim as it regards how we come to know Ψ, i.e. the statements about thin objects. As discussed in §2, according to the neo-logicist program, abstraction principles are *the* means by which we come to know about thin objects. Therefore, abstraction principles must give a full epistemological. The second claim fails to do this; assuming it is true, it only gets as far as guaranteeing the possibility of knowledge. It says nothing of how we actually acquire that knowledge.

Therefore, I believe the asymmetric view, despite aiming lower, must attempt to justify the stronger claim “If $\Phi \Rightarrow \Psi$ and $\mathcal{K}(\Phi)$, then $\mathcal{K}(\Psi)$.” This claim, however, is susceptible to a counterexample of identical form to the symmetric view. For HP, that is a non-problematic case where someone knows that two collections of things are equinumerous, but does not know that the two collections have the same number. Regardless, we will return to the weaker condition later.

4 | Pirahã: Equinumerosity Without Numbers

The Pirahã are an indigenous tribe that lives within Amazon rainforest. Despite regular contact with Brazilian merchants and external researchers, the Pirahã remain a monolingual community (“What Does Pirahã” Everett). There are only four fluent or near fluent speakers of Pirahã outside of the core tribe: Daniel L. Everett, Keren Madora, Steven Neil Sheldon, and José-Augusto Diarroi-Pirahã (“What Does Pirahã” Everett).

The Pirahã were thrust into the forefront of modern linguistic debate upon Daniel Everett’s publication of *Cultural Constraints on Grammar and Cognition in Pirahã* in 2005. In this paper, Everett outlines a litany of surprising (missing) features of the Pirahã language: he claims the Pirahã lack [1] number words or use of numbers, [2] quantificational words, [3] abstract color words, [4] relative tenses, [5] creation myths and fiction, [6] memory of anything preceding two generations back, and, most controversially, [7] any recursion or “embedding.” While linguistics is mainly concerned with [7] as it denies Noam Chomsky’s Universal Grammar, I am most concerned here with Pirahã's lack of numbers and how it relates to HP. I am also tangentially interested in [2], [3], and [5] as they likewise deal with a level of abstraction.

The best demonstration of the Pirahã's number capabilities (or lack thereof) comes in later papers in collaboration with Everett. There are two particularly instructive experiments from Frank et al.. The first was broken into two parts and aim to identify possible number words. In the first part of the experiment, the researchers placed increasingly large sets of spools of thread on the table in front of Pirahã men and women. At each set the participants were asked "how much/many is this? (Translated into Pirahã by [Everett])" (820). Conversely, in the second part, the Pirahã were presented with consecutive sets of decreasing size and asked the same question. To all of these questions, the participants only used the words *hói, hoí, and baágiso*. For the sets of increasing size, *hói* was used exclusively and solely for sets containing just one spool; only *hoí* was used for sets of two spools while *hoí and baágiso* were both used for sets ranging from 3-10 spools (820). For sets of decreasing size, *hói* was used for sets ranging from 1-6 spools, *hoí* for sets of 4-10 spools, and *baágiso* for sets of 7-10 spools (820). This experiment indicates that the Pirahã have no actual number words, not even for one. The most promising word, *hói*, was used for both sets of 1 and 6 spools. Instead, *hói, hoí, and baágiso* most likely refer to relative sizes: this set is smaller than the that set; that set is much larger than this other set.

The second experiment aimed to determine if Pirahã still retained the ability to create equinumerous sets (what they call "numerical cognition") despite lacking number words (821). The Pirahã were presented with sets of spools sizes 1-10 and asked to place on the table an equivalent amount of uninflated rubber balloons.[1] They repeated this experiment in multiple ways. In some trials, the spools were evenly placed, unevenly placed, hidden behind a folder after the Pirahã had a moment to observe, etc.. For both even and uneven placement, the Pirahã had "nearly perfect" performance (822). In the other trials, the Pirahã struggled and especially struggled for sets of larger sizes (822). However, overall, the Pirahã demonstrated a clear understanding of equinumerosity.

These two results do not yet provide a meaningful counterexample to HP. While it's clear that the Pirahã understand equinumerosity, experiment one does not prove that the Pirahã do not apprehend numbers. It merely shows that their language does not have numerical machinery. Thus, the neo-logicist can, so far, insist that the Pirahã still internally know of numbers but they just don't have the tools to express it. In what follows, I attempt to show that the Pirahã do not know of numbers *at all* and that they are not simply making a mistake.

In Everett's aforementioned *Cultural Constraints…* he recounts his and Karen Madora's (his ex-wife) attempts to teach the Pirahã numbers:

> "In 1980, at the Pirahã's urging, my wife and I began a series of evening classes in counting and literacy. My entire family participated, with my three children (9, 6, and 3 at that time) sitting with Pirahã men and women and working with them. Each evening for eight months my wife would try to teach Pirahã men and women to count to ten in Portuguese. They told us that they wanted to learn this because they knew that they did not understand nonbarter economic relations and wanted

[1] This was usually done by matching one balloon to one spool across the set. This does not invalidate any results. First, if done correctly, it demonstrates understanding of equinumerous sets of size one. Second, it resembles the bijective functions used to determine equinumerosity between infinite sets; we don't immediately apprehend the size of infinite sets, but instead create a process that corresponds one item of a set to one item of the other set.

> to be able to tell whether they were being cheated. After eight months of daily efforts, without ever needing to call them to come for class (all meetings were started by them with much enthusiasm), the people concluded that they could not learn this material, and classes were abandoned. Not one learned to count to ten, and not one learned to add 3 + 1 or even 1 + 1 (if regularly responding "2" to the latter is evidence of learning)—only occasionally would some get the right answer" (625-626).

This story refutes the neo-logicist's defense of lacking numerical machinery. *At their request,* an extensive and serious attempt was made to provide the Pirahã with the language to refer to numbers. They had very good reason to want to grasp numerical concepts and much enthusiasm for eight months. If they apprehended numbers, they had every reason and tool to express it.

However, it is Everett's description of why the Pirahã failed to convey understanding that is the most convincing. He continues

> "It should be underscored here that the Pirahã ultimately not only do not value Portuguese (or American) knowledge but oppose its coming into their lives… If one tries to suggest (as we originally did, in a math class, for example) that there is a preferred response to a specific question, they will likely change the subject and/or show irritation" (626).

They very idea that a singular term, i.e. "one" or "two," should refer to a specific property of a set is incommensurate with the Pirahã's perception of the world. It is clear that this could not be because the Pirahã lack notions of correctness and incorrectness. The Pirahã, obviously, regularly identify thick objects correctly and notify Everett when he spoke Pirahã incorrectly.

Thus, it must be because the Pirahã are missing the mathematical content referred to in mathematical statements. It would be like asking a blind person "is blue complementary to orange?" There is a correct answer, but they lack understanding of the content to judge it. Moreover, this explains the Pirahã's frustration. The sighted person's continued insistence of a correct answer would evidently be quite irritating and incomprehensible to the blind person. The acceptance of Everett's and, by extension, Portuguese/American civilization's claims regarding 'numbers' would require the Pirahã to have faith in or grant epistemic authority to foreigners (an understandably unappealing decision.)

It is because of this specific attitude towards mathematical content that I believe the Pirahã internally fail to grasp/know of/understand/perceive numerical objects, not because they merely fail to outwardly express it. Then, to be explicit, because Pirahã have been shown to not grasp numbers, they cannot make sense of the claim that $\#\alpha = \#\beta$, and hence, the Pirahã cannot know that $\#\alpha = \#\beta$. Therefore, because the Pirahã demonstrate understanding of equinumerosity but fail to perceive numbers, they present a legitimate counterexample to the epistemic component of HP. This in turn places a shadow on the plausibility of neo-logicist metaphysics.

5 | Daniel L. Everett

Since the publication of *Cultural Constraints on Grammar and Cognition in Pirahã* Everett has been harshly disparaged by Noam Chomsky, Steven Pinker, and more. Though this largely surrounds his discussion of recursion in Pirahã and how culture affects grammar, there is some criticism of his claims around numbers, e.g. Nevins et al.. However, it uses outdated data and is largely rebuked by Everett in *Pirahã Culture and Grammar: A Response to Some Criticisms.* More than anything though, I accept his claim that the Pirahã lack number words and all that follows directly because of [1], in my view, sufficient consistency between work done by

Gordon in 2004, Everett, and Frank et al. in 2008 and [2] lack of criticism from the three other fluent or near-fluent bilingual speakers.

6 | Objections and Replies

Finally, I want to return to the weaker claim Linnebo puts forward for HP: If $\alpha \approx \beta \Rightarrow \#\alpha = \#\beta$ and $\mathcal{K}(\alpha \approx \beta)$, then $\Diamond\mathcal{K}(\#\alpha = \#\beta)$. The neo-logicist, if convinced by my counterexample to the stronger claim, most sensibly would take refuge in the weaker asymmetric claim. My counterexample fails to immediately pose a threat to this configuration; failure in one instance to obtain knowledge of numbers from knowledge of equinumerosity does not undermine the possibility claim.

To this I concede. This counterexample cannot provably deny this weaker version of HP. However, the case of the Pirahã does propose a difficult challenge for this weak HP. As I argued in §4, the Pirahã are not making false statements about numbers (e.g. claiming two equinumerous sets have different numbers). The Pirahã simply *are not* making statements about numbers and *are not* perceiving numbers. This failure to achieve knowledge is wholly distinct from incorrect assessments. Incorrect numerical assessments of sets may come from confusion about equinumerosity or competing external info (e.g. Alice is persuaded by an angry logician), but nonetheless retain a baseline conception of numbers. As a result, failures can easily be resolved by reappreciating the facts of the matter. For the Pirahã, knowledge cannot be easily instated as significant portions of the facts are inaccessible. Simply telling them that this set has a corresponding number and that set has the same corresponding number does not seem to suffice for the Pirahã; they do not take our claims on faith and they demand a more direct and intimate understanding of ‘number-ness’ to agree.[2]

It is this inaccessibility that provides a direct challenge to weak HP. Abstraction principles and specifically HP, by their definition, are intended to directly describe, to quote Frege again, “how… numbers are given to us” (73). It is supposed that by the recognition of the unity relation ‘equinumerosity,’ we come to know of the thin objects that are numbers. Because the Pirahã, for whatever reason, do not complete this move, even if the neo-logicist accepts weak HP, they have far more work to do in explicating how and under what circumstances we actually do come to recognize thin objects. If even possible, this prospect looks dangerous. If that explication turns out to be cultural constraints (as Everett would suggest) or some certain brain configuration, it seems to me, the neo-logicist risks offloading work onto mental constructions and losing the platonic purity of numbers initially desired.

7 | Conclusion

The symmetric approach appears untenable; it fails wholesale to justify commitment [1]. HP does not simply “recarve” the same content, there is evidently something more to numbers than equinumerosity. In this light, any symmetric abstraction principle seems precarious. In the end, though, I do not conclude HP and the neo-logicist program dead in the face of current data of Pirahã cognition. The weak asymmetric approach has not been thoroughly refuted; in its current incarnation it fails to justify commitment [1]. For it to succeed, though, it must provide a clearer story on the acquisition of thin objects that is not reliant on cultural or mental constructions.

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[2] This is understandable. Part of the appeal common of HP is how clearly it strikes most people. Our intimate intuition of a number and numbers seem obviously commensurate with equinumerosity.

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